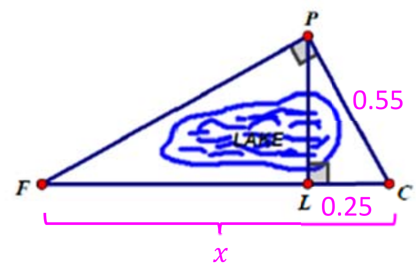
Formal Geometry Name________________________________

Ch 8 Review Worksheet #1

The student may find it helpful to look at the Trigonometry Handbook and the Trigonometry page on www.mathguy.us for assistance with Trigonometry.

1) In the diagram shown, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 miles from the lifeguard chair. The straight paths from both the campground and first stations to the park ranger station are perpendicular. If the path to the park ranger station to the campground is 0.55 miles, Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Show work to provide evidence for your conclusion.



If you can get past the English on this one, you can mark the segments as I have in the diagram to the right. Then,

A side squared = the product of the part of the base corresponding to it and the entire base (see page 5 of this document).

$$(0.55)^2 = 0.25x$$

$$x = \frac{(0.55)^2}{0.25} = 1.21$$

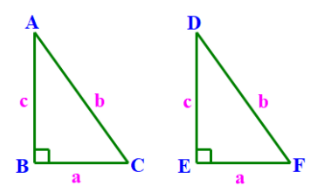
So, the distance from the First Aid Station to the Campground is 1.21 miles.

Gerald is not correct.

2) Given $\Delta ABC \sim \Delta DEF$ with $\angle B$ and $\angle E$ right angles, select **all** expressions that are equivalent to cos A.

A) cos B
B) sin A
C) cos C
D) sin C
E) cos D
F) sin D
G) cos F
H) sin F
I) sin E

I have employed a little trick here to approach this question with logic and confidence.



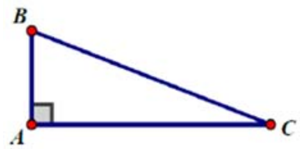
Congruent triangles are similar triangles with the same side lengths. So, in the diagram to the right, I have created congruent triangles with this in mind. Since we are only dealing with the angles of the two triangles, not the lengths, this trick works.

Based on the diagram, we can see that:

$\cos A = \frac{c}{b} = \sin C = \cos D = \sin F$. These are the only function values equal to $\frac{c}{b}$.

Looking these up in the above list, we see that the correct solutions are: **D, E, H**

3) In right $\Delta ABC, m\angle B \neq m\angle C$. Let $\sin B = r$ and $\cos B = s$. Write an expression that represents $\sin C - \cos C$ in terms of r and s.



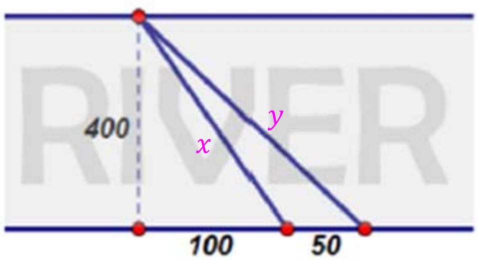
$\sin B = r = \cos C$

$\cos B = s = \sin C$

Then,

$\sin C - \cos C = \boldsymbol{s - r}$

4) Charlie wants to swim across a river that is 400 meters wide. He begins swimming perpendicular to the shore he started from, but he ends up on the opposite side 100 meters down river from where he started due to the current. Carter starts swimming from the same spot as Charlie and ends up 50 meters further down river than Charlie. How much farther did Carter swim? Round your answer to the nearest meter.



If we add x and y to the diagram, as shown to the left, this question is asking for $y - x$.

Using the Pythagorean Theorem,

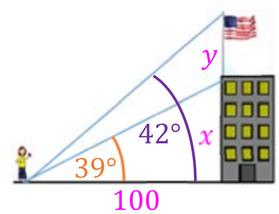
$x^2 = 400^2 + 100^2 = 170{,}000 \quad \rightarrow \quad x = 412.31$ m

$y^2 = 400^2 + 150^2 = 182{,}500 \quad \rightarrow \quad y = 427.20$ m

$y - x \approx 427.20 - 412.31 \approx$ **15 m**

For #5 – 6: A flagpole is at the top of a building. A person is standing 100 feet from the base of the building. The angle of elevation to the top of the pole is 42°, and the angle of elevation to the bottom of the pole is 39°.

5) Find the height of the building to the nearest foot.



The height of the building is x in the diagram to the right. Based on the smaller triangle, we have:

$$\tan 39° = \frac{x}{100}$$

$\boldsymbol{x} = 100 \cdot \tan 39° \approx$ **81 ft**

6) Find the length of the flagpole to the nearest foot.

The height of the building plus the flagpole is $(x + y)$ in the diagram above. Based on the larger triangle, we have:

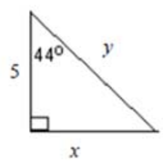
$$\tan 42° = \frac{(x + y)}{100}$$

$(x + y) = 100 \cdot \tan 42° \approx 90$ ft

The height of the flagpole is: $\boldsymbol{y} = (x + y) - x \approx 90 - 81 \approx$ **9 ft**

For #7 – 10, find the variable(s), rounded to the nearest hundredth.

7)



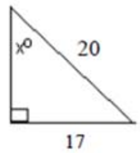
$$\tan 44° = \frac{x}{5}$$

$$\boldsymbol{x} = 5 \cdot \tan 44° \approx \mathbf{4.83}$$

$$\cos 44° = \frac{5}{y}$$

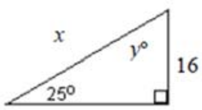
$$\mathbf{y} = \frac{5}{\cos 44°} \approx \mathbf{6.95}$$

8)



$$\boldsymbol{x} = \sin^{-1}\frac{17}{20} \approx \mathbf{58.21°}$$

9)



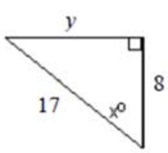
$$\sin 25° = \frac{16}{x}$$

$$\boldsymbol{x} = \frac{16}{\sin 25°} \approx \mathbf{37.86}$$

$$y + 25° = 90°$$

$$\mathbf{y} = 90° - 25° \approx \mathbf{65°}$$

10)



$$x = \cos^{-1}\frac{8}{17}$$

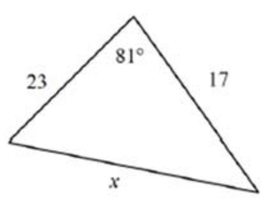
$$\boldsymbol{x} \approx \mathbf{61.93°}$$

$$8^2 + y^2 = 17^2$$

$$y^2 = 17^2 - 8^2 = 225$$

$$\boldsymbol{y} = \mathbf{15}$$

11) Find x to the nearest hundredth.



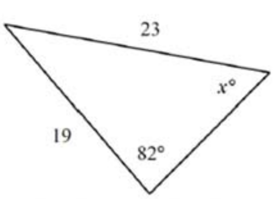
Law of Cosines:

$$x^2 = 17^2 + 23^2 - (2 \cdot 17 \cdot 23) \cdot \cos 81°$$

$$x^2 = 695.668$$

$$\boldsymbol{x} = \mathbf{26.38}$$

12) Find x to the nearest hundredth.

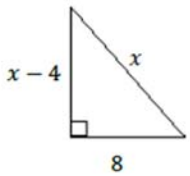


Law of Sines:

$$\frac{\sin x}{19} = \frac{\sin 82°}{23} \quad \rightarrow \quad \sin x = \frac{19 \cdot \sin 82°}{23} = 0.81805$$

$$x = \sin^{-1}(0.81805) = \mathbf{54.89°}$$

13) Solve for x. No calculator!

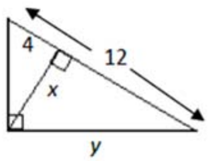


$$(x-4)^2 + 8^2 = x^2$$

$$x^2 - 8x + 16 + 64 = x^2$$

$$80 = 8x \quad \rightarrow \quad \boldsymbol{x = 10}$$

14) Find x and y. No calculator!



See chart on next page.

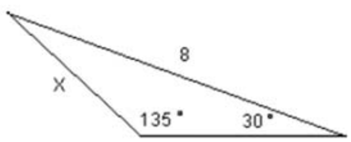
$x^2 = 4(12-4) = 32$

$\boldsymbol{x} = \sqrt{32} = \boldsymbol{4\sqrt{2}}$

$y^2 = (12-4)\cdot 12 = 96$

$\boldsymbol{y} = \sqrt{96} = \boldsymbol{4\sqrt{6}}$

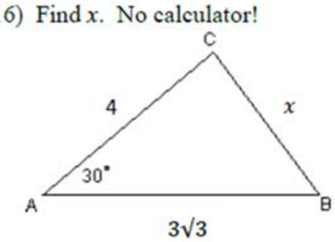
15) Find x. No calculator!



Law of Sines:

$$\frac{\sin 30°}{x} = \frac{\sin 135°}{8} \quad \rightarrow \quad \boldsymbol{x} = \frac{8\cdot\sin 30°}{\sin 135°} = \frac{8\cdot\frac{1}{2}}{\frac{\sqrt{2}}{2}} = \boldsymbol{4\sqrt{2}}$$

16) Find x. No calculator!



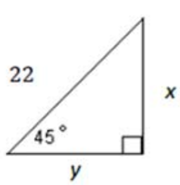
Law of Cosines:

$$x^2 = (3\sqrt{3})^2 + 4^2 - \left(2\cdot 3\sqrt{3}\cdot 4\right)\cdot\cos 30°$$

$$x^2 = 27 + 16 - 24\sqrt{3}\cdot\frac{\sqrt{3}}{2} = 43 - 36 = 7$$

$$\boldsymbol{x = \sqrt{7}}$$

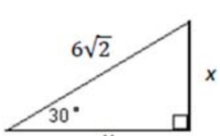
17) Find x and y. No calculator.



$$\boldsymbol{x} = \frac{22}{\sqrt{2}} = \frac{22}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{22\sqrt{2}}{2} = \boldsymbol{11\sqrt{2}}$$

$$\boldsymbol{y} = x = \boldsymbol{11\sqrt{2}}$$

18) Find x and y. No calculator.

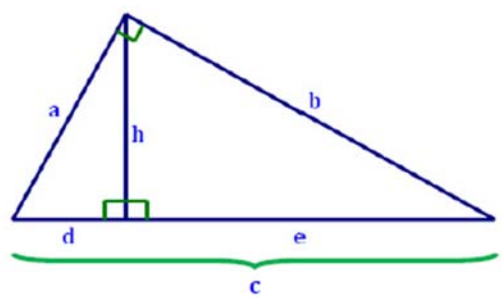


$$\boldsymbol{x} = 6\sqrt{2} \div 2 = \boldsymbol{3\sqrt{2}}$$

$$\boldsymbol{y} = \sqrt{3}\cdot 3\sqrt{2} = \boldsymbol{3\sqrt{6}}$$

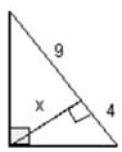
For # 19 – 21: Solve for the variables (no calculator). If needed, leave answers in radical form.

From the Geometry Handbook,



From the two inside triangles	From the inside triangle on the left and the outside triangle	From the inside triangle on the right and the outside triangle
$\frac{h}{d} = \frac{e}{h}$ or $h^2 = d \cdot e$	$\frac{a}{d} = \frac{c}{a}$ or $a^2 = d \cdot c$	$\frac{b}{e} = \frac{c}{b}$ or $b^2 = e \cdot c$
The height squared = the product of: the two parts of the base	The left side squared = the product of: the part of the base below it and the entire base	The right side squared = the product of: the part of the base below it and the entire base

19.



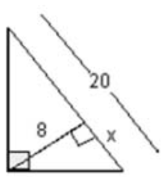
From the information above:

The height squared = the product of the two parts of the base.

$$x^2 = 4 \cdot 9 = 36$$

$$\boldsymbol{x} = \sqrt{36} = \mathbf{6}$$

20.



From the information above:

The height squared = the product of the two parts of the base.

$$8^2 = x(20 - x)$$

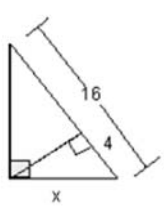
$$64 = 20x - x^2$$

$$x^2 - 20x + 64 = 0$$

$$(x - 4)(x - 16) = 0$$

$\boldsymbol{x} = \mathbf{4, 16}$ Notice that both solutions are valid, and that they add to 20.

21.



From the information above:

A side squared = the product of the part of the base corresponding to it and the entire base.

$$x^2 = 4 \cdot 16 = 64$$

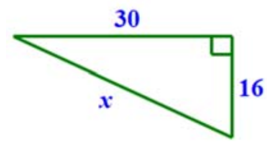
$$\boldsymbol{x} = \sqrt{64} = \mathbf{8}$$

22. If Captain Jack Sparrows' treasure map reads that to find the treasure ye must walk 6 paces north from the stump, 12 paces west, 10 paces north, and 18 paces west. How far from the stump is the treasure?

In total, Jack goes 16 paces north and 30 paces west.

$x^2 = 16^2 + 30^2 = 1156$

$\boldsymbol{x} = \sqrt{1156} = \mathbf{34}$ **paces**

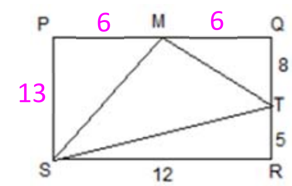


23. Classify a triangle with sides of length 5, 11, and 13 as acute, right, or obtuse.

$5^2 + 11^2 = 146 \quad 13^2 = 169$

$146 < 169$, so the triangle is **obtuse.**

24. M is the midpoint of a side of rectangle PQRS.
Find the perimeter of ΔMST. No calculator.



$ST = \sqrt{5^2 + 12^2} = 13$

$TM = \sqrt{6^2 + 8^2} = 10$

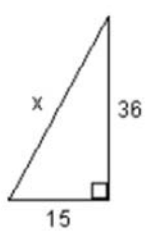
$MS = \sqrt{6^2 + 13^2} = \sqrt{205}$

$\boldsymbol{P}(\Delta \boldsymbol{MST}) = ST + TM + MS = 13 + 10 + \sqrt{205} = \mathbf{23} + \sqrt{\mathbf{205}}$

For #25 – 39. Solve for the variables.

Problems #25-27 are solved with the Pythagorean Theorem: $a^2 + b^2 = c^2$

25.



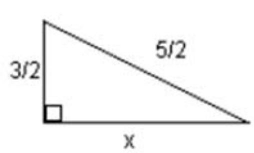
$x^2 = 15^2 + 36^2$

$x^2 = 225 + 1296$

$x^2 = 1521$

$\boldsymbol{x} = \mathbf{39}$

26.



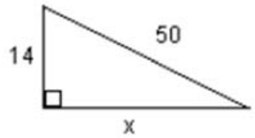
$x^2 + \left(\frac{3}{2}\right)^2 = \left(\frac{5}{2}\right)^2$

$x^2 + \frac{9}{4} = \frac{25}{4}$

$x^2 = \frac{16}{4} = 4$

$\boldsymbol{x} = \mathbf{2}$

27.



$x^2 + 14^2 = 50^2$

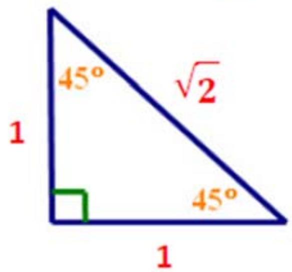
$x^2 + 196 = 2500$

$x^2 = 2304$

$\boldsymbol{x = 48}$

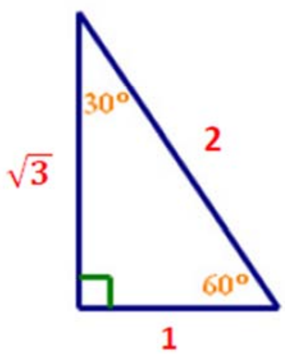
From the Geometry Handbook (for Problems #28-39):

45°-45°-90° Triangle



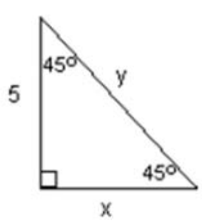
In a **45°-45°-90° triangle**, the congruence of two angles guarantees the congruence of the two legs of the triangle. The proportions of the three sides are: $\mathbf{1 : 1 : \sqrt{2}}$. That is, the two legs have the same length and the hypotenuse is $\sqrt{2}$ times as long as either leg.

30°-60°-90° Triangle



In a **30°-60°-90° triangle**, the proportions of the three sides are: $\mathbf{1 : \sqrt{3} : 2}$. That is, the long leg is $\sqrt{3}$ times as long as the short leg, and the hypotenuse is **2** times as long as the short leg.

28.

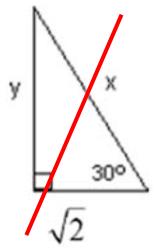


The sides have the same length.

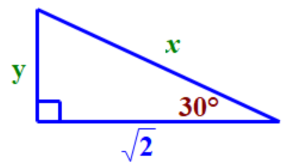
$\boldsymbol{x = 5}$

$y = \sqrt{2} \cdot 5 = \boldsymbol{5\sqrt{2}}$

29.



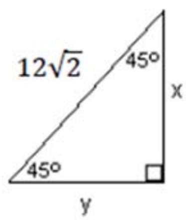
This triangle is drawn to trick us. Let's re-draw it.



$$y = \sqrt{2} \div \sqrt{3} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$x = 2 \cdot \frac{\sqrt{6}}{3} = \frac{2\sqrt{6}}{3}$$

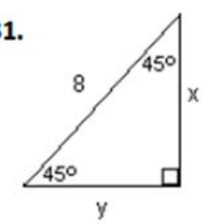
30.



$$x = 12\sqrt{2} \div \sqrt{2} = 12$$

$$y = x = 12$$

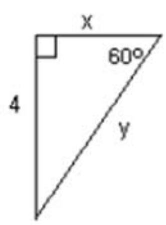
31.



$$x = 8 \div \sqrt{2} = \frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$y = x = 4\sqrt{2}$$

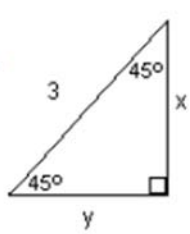
32.



$$x = 4 \div \sqrt{3} = \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

$$y = 2 \cdot \frac{4\sqrt{3}}{3} = \frac{8\sqrt{3}}{3}$$

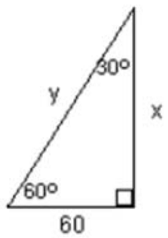
33.



$$x = 3 \div \sqrt{2} = \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$y = x = \frac{3\sqrt{2}}{2}$$

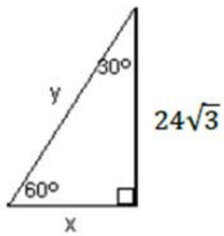
34.



$x = 60 \cdot \sqrt{3} = \mathbf{60\sqrt{3}}$

$y = 2 \cdot 60 = \mathbf{120}$

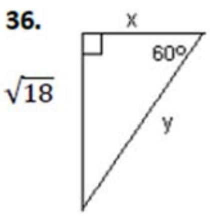
35.



$x = 24\sqrt{3} \div \sqrt{3} = \mathbf{24}$

$y = 2 \cdot 24 = \mathbf{48}$

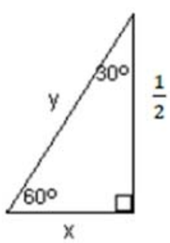
36.



$x = \sqrt{18} \div \sqrt{3} = \sqrt{\mathbf{6}}$

$y = 2 \cdot \sqrt{6} = \mathbf{2\sqrt{6}}$

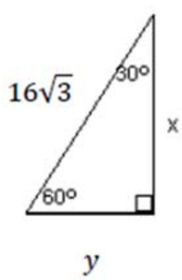
37.



$$x = \frac{1}{2} \div \sqrt{3} = \frac{1}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{\mathbf{3}}}{\mathbf{6}}$$

$$y = 2 \cdot \frac{\sqrt{3}}{6} = \frac{\sqrt{\mathbf{3}}}{\mathbf{3}}$$

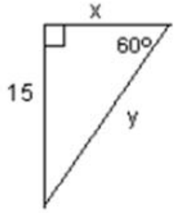
38.



$y = 16\sqrt{3} \div 2 = \mathbf{8\sqrt{3}}$

$x = 8\sqrt{3} \cdot \sqrt{3} = 8 \cdot 3 = \mathbf{24}$

39.



$$x = 15 \div \sqrt{3} = \frac{15}{\sqrt{3}} = \frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = \mathbf{5\sqrt{3}}$$

$y = 2 \cdot 5\sqrt{3} = \mathbf{10\sqrt{3}}$